INSTITUT UCAC-ICAM

Entrance Examination – August 2021

To be filled by the candidate:	Reserved for the Institut
Name: ………………………………..…… Surname: ……………….…………. Examination center: ………………… Seat N°: ……….. Subject : ………………………………………………………………………	*Anonymous N°:* ………..

Reserved for the Institut	✓	Reserved for the Institut
Score:	1^{ST} CYCLE OF TRAINING **Mathematics Test Questions**	*Anonymous N°:* ………..

INSTRUCTIONS: This paper is made up of 40 questions. Each question carries 4 suggested answers. Select the most appropriate answer and mark it on the answer sheet provided.

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1. Let p be the statement "James works hard" and q the statement "James will be successful". The contrapositive of the statement "$p \Rightarrow q$" is:
   A. $\sim q \Rightarrow p$
   B. $\sim p \Rightarrow \sim q$
   C. $q \Rightarrow p$
   D. $\sim q \Rightarrow \sim p$

2.

$$\lim_{x\to 0}\left(\frac{|x|}{x}\right) =$$

   A. 1
   B. ∞
   C. 0
   D. $\nexists$ (Does not exist)

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3. The partial fraction form of

$$\frac{2x+1}{x^3-1}, \qquad \text{is:}$$

   A. $\frac{a}{x-1} + \frac{bx+c}{x^2+x+1}$
   B. $a + \frac{b}{x-1} + \frac{cx+d}{x^2-x+1}$
   C. $a + \frac{b}{x-1} + \frac{cx+d}{x^2+x+1}$

D. $\frac{a}{x-1}+\frac{b}{(x-1)^2}+\frac{c}{(x-1)^3}$

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4. The parametric equation o the rectangular hyperbola $xy + x = c^2$ is:
   A. $x = ct; y = \frac{c}{t} - 1$
   B. $x = 1 + ct; y = \frac{c}{t}$
   C. $x = ct; y = \frac{c}{t}$
   D. $x = ct; y = \frac{c}{t} + 1$

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6. Given that

$$f(2x) = \frac{1}{x}, x \neq 0,$$

Then $f\left(\frac{1}{x}\right) =$

A. $\frac{1}{x}$
B. $\frac{x}{2}$
C. $\frac{2}{x}$
D. $2x$

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5. Given that

$$f(x) = \begin{cases} x^2, & \text{for } 0 \leq x \leq 1 \\ 3x - 2, & \text{for } 1 < x \leq 3 \\ 9 - x, & \text{for } x > 3 \end{cases}$$

   For which value of x is f NOT continuous?
   A. 2
   B. 3
   C. 1
   D. 0

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6. The mean value of the function e^{-2x} in the interval $0 \leq x \leq 2$ is:
   A. $\frac{1}{2}(1 - e^{-4})$
   B. $-\frac{1}{4}e^{-4}$
   C. $1 - e^{-4}$
   D. $\frac{1}{4}(1 - e^{-4})$

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7. The general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 3y = 0,$$

Where A, B and ε are constants, is:

A. $A\cos 4x + B\sin x$
B. $Ae^x\cos(4x + \varepsilon)$
C. $Ae^x + Be^{-4x}$
D. $(Ax + B)e^{-4x}$

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8. $\operatorname{arctanh}(x) =$

A. $\ln\left(x + \sqrt{x^2 + 1}\right)$, for all x
B. $\ln\left(x + \sqrt{x^2 + 1}\right)$, for $-1 < x < 1$
C. $\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$, for $-1 \le x \le 1$
D. $\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, for $-1 < x < 1$

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9. The moment of inertia, I_G, of a solid sphere, of mass m and radius a, about its center is $\frac{2}{5}ma^2$. The moment of inertia of the sphere about an axis through its tangent is:

A. $\frac{4}{3}ma^2$
B. ma^2
C. $\frac{3}{5}ma^2$
D. $\frac{7}{5}ma^2$

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10. Given that

$$f(x) = \begin{cases} x^2 - 9, & x \ne 3 \\ x - 3, & x = 3 \end{cases}$$

Which of the following is true about f?

A. f is continuous at $x = 3$
B. f is not defined at $x = 3$
C. $\lim_{x\to 3} f(x) = 3$
D. f is defined at $x = 3$

11.

$$\int_1^e \frac{2\ln x}{x}\,dx =$$

A. 1
B. 3
C. 2
D. e

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12. If ae^{-4x} is a particular integral of the integral of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=5e^{-4x}$$

Then the value of a is:
A. 2
B. $\frac{1}{2}$
C. $-\frac{1}{2}$
D. -2

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13. The Cartesian equation of the midpoint M of $P\left(4t,\frac{1}{t}\right)$ and $Q\left(t,\frac{4}{t}\right)$ as t varies is:
A. $xy=1$
B. $xy=16$
C. $xy=25$
D. $4xy=25$

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14. In which quadrant does the following system

$$\begin{cases} y=2x+1 \\ y-1=\frac{1}{2}(x-2) \end{cases}, \qquad \text{contain a solution?}$$

A. First
B. Second
C. Third
D. Forth

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15. The negation of the statement "Some children are not tall" is:
A. All children are tall.
B. No child is tall.
C. Not all children are tall.
D. All children are tall.

16.

$$\lim_{n\to\infty}\left[\sum_{k=1}^{n}\left(-\frac{1}{2}\right)^{k}\right]=$$

A. $-\frac{1}{3}$
B. 0
C. 1
D. ∞

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17. If $g(x) = [x]$ demotes the greatest integer function, and that $f(x) = 1 + g(x)$, then

$$\int_0^3 f(x)\,dx =$$

A. 3
B. 7
C. 2
D. 4

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18. A particle P moves with uniform angular speed of 2 rads^{-1} along a curve whose polar equation is given by $r = \sin 2\theta$. The magnitude of the transverse component of the acceleration of P when $\theta = \frac{\pi}{6}$ is:

A. 16 ms^{-2}
B. 18 ms^{-2}
C. 10 ms^{-2}
D. 8 ms^{-2}

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19. The value of k for which the function f is continuous, at $x = 0$, where

$$f(x) = \begin{cases} \dfrac{\sin 3x}{x}, x < 0 \\ \dfrac{x^2 + k}{1 - k}, x \geq 0 \end{cases}, \qquad \text{is:}$$

A. $\frac{3}{4}$
B. $\frac{4}{3}$
C. $\frac{1}{2}$
D. $\frac{3}{2}$

---

20. Given that

$$f(x) = \frac{x-2}{x^2+4}$$

The number of asymptotes in the graph of $y = f(x)$ are:

A. 1
B. 0
C. 2
D. 3

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21. If f is an odd function, then,

$$\int_{-a}^{a} f(x)\,dx =$$

A. 0
B. 1
C. $\int_0^a f(x)\,dx$
D. $2\int_0^a f(x)\,dx$

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22. The area under the curve with parametric equations $x = 1 + t^2, y = t(2-t)$ in the interval $0 \le t \le 1$ is:

A. $\frac{5}{4}$
B. $\frac{5}{6}$
C. $\frac{6}{5}$
D. $\frac{2}{3}$

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23. The period of the damped harmonic motion defined by

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0, \qquad \text{is:}$$

A. $\frac{1}{2}\pi$
B. π
C. 2π
D. 4π

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24. The center of gravity of a right angled triangle OAB with right angle at O is:

A. $\left(\frac{2}{3}OA,\ \frac{2}{3}OB\right)$
B. $\left(\frac{1}{3}OA,\ \frac{1}{3}OB\right)$
C. $\left(\frac{1}{2}OA,\ \frac{1}{2}OB\right)$

D. $\left(\frac{2}{3}OA,\ \frac{1}{3}OB\right)$

25.

$$\cosh(\ln 4) =$$

A. $\frac{17}{4}$
B. $\frac{15}{4}$
C. $\frac{17}{8}$
D. $\frac{15}{8}$

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26. A function $f(x)$ has a limit at $x = a, a \in D_f$ is:
A. $\lim_{x \to a} f(x) = f(a)$
B. $\lim_{x \to a^+} f(x) = f(a)$
C. $\lim_{x \to a^-} f(x) = f(a)$
D. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

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27. The energy equation of a compound pendulum is $2a\,\dot{\theta}^2 = 3g\cos\theta$. The period of small oscillations of the pendulum is:
A. $4\pi\sqrt{\frac{a}{3g}}$
B. $2\pi\sqrt{\frac{2a}{3g}}$
C. $\sqrt{\frac{3a}{g}}$
D. $\sqrt{\frac{4a}{3g}}$

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28. If $\omega = iz$ is a transformation from the $z -$ complex plane to the $\omega -$ complex plane, then the invariant point is:
A. -1
B. 1
C. i
D. 0

29.

$$\int_1^e \frac{2\ln x}{x}\,dx =$$

A. 3
B. 1
C. 2
D. e

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30. Which of the following is **not** true about the graph of$y = f(x)$?
   A. f is a function if and only if a vertical line drawn on the graph intersects the graph of $y = f(x)$ at not more than one point.
   B. f is function if and only if a horizontal line drawn on the graph intersects the graph of $y = f(x)$ at not more than one point.
   C. The graph of $y = f(x)$ can intersect a horizontal asymptote.
   D. f is injective if a horizontal line drawn on the graph intersects the graph $y = f(x)$ at not more than one point

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31. A particle P moves with constant angular speed ω on a curve with polar equation$r = ae^{\theta}$. The transverse component of the velocity of P is:
   A. $a\omega e^{\theta}$
   B. $a\omega$
   C. ωe^{θ}
   D. ae^{θ}

---

32. The equation of a curve is given parametrically as $x = a\sec\theta\,, y = b\tan\theta$. The Cartesian equation of this curve is given by:
   A. $y^2 = 4abx$
   B. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
   C. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
   D. $x^2 - y^2 = (ab)^2$

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33. Given that

$$I_n = \int_0^2 \sqrt{x}(2-x)^{n+\frac{1}{2}}\,dx$$

Using the substitution $x - 1 = \sinh u$ gives $I_0 =$

A. $\int_{0}^{\frac{\pi}{2}}(1-\sinh u)\,du$
B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\sinh u)\,du$
C. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos^2 u)\,du$
D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos u)\,du$

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34. The solution to the equation

$$\tanh^{-1}\left(\frac{x^2-1}{x^2+1}\right)=\ln 2\,,\qquad \text{is:}$$

A. $\ln 2$
B. 2
C. ± 2
D. -2

35.

$$\frac{d}{dx}(\ln\cosh 2x)=$$

A. $\frac{1}{\cosh 2x}$
B. $\frac{2}{\sin 2x}$
C. $\frac{1}{2}\tanh 2x$
D. $2\tanh 2x$

---

36. Given that

$$\int_{a}^{4}\frac{x+6}{x}\,dx=3+12\ln 2$$

Then the value of a is:

A. 1
B. 6
C. 2
D. 3

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37. During a direct impact between two spheres, if e is the coefficient of restitution, then the collision is said to be perfectly inelastic if:

A. $e = 1$
B. $e = 0$
C. $0 < e < 1$
D. $0 \leq e \leq 1$

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38. The differential equations

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + n^2x = 0$$

Represents oscillatory motion when:
A. $k^2 > n^2$
B. $k^2 < n^2$
C. $k^2 = n^2$
D. $4k^2 > n^2$

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39. The moment of inertia of an object of mass $2m$ is $\frac{8ma^2}{3}$. Its radius of gyration is:
A. $\frac{4a^2}{3}$
B. $\frac{\sqrt{3}}{2a}$
C. $\frac{2a}{\sqrt{3}}$
D. $\frac{\sqrt{3}}{3}a$

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40. Given that $i = \sqrt{-1}$ then $i^{37} =$
A. i
B. -1
C. $-i$
D. 1
